Some observations are lastly added concerning the nature and situation of the ciliary processes in various animals; also on the nature of the marsupium nigrum of birds, and the horseshoe-like appearance in the choroid of fishes ; both which have improperly been termed muscular,—the former being a mere duplicature of a membrane which may be unfolded ; and in the latter the whole mass being evidently of an uniform texture, the fibrous appearance which has misled some former observers being the effect of transverse fissures, or cracks, which may easily be mistaken for filaments.

The lecture concludes with a few observations on the bony scales of the eyes of birds, to which the author denies any concern in changing the focus of the eye ; and on a cavity observable in the eyes of some insects which has been supposed to be in some measure subservient to this purpose.

On the necessary Truth of certain Conclusions obtained by Means of Imaginary Quantities. By Robert Woodhouse, *A.M. Fellow of Caius College. Communicated by the Rev.* S. Vince, *A.M. Plumian Professor of Astronomy in the University of Cambridge.* Read January 8, 1801. [*Phil. Trans.* 1801, *p.* 89.]

The object of this paper is to show, that we may be assured of the justness and accuracy of conclusions obtained by means of imaginary quantities, without verifying such conclusions by separate investigations, or without inferring their truth from analogy. In the first part the author premises at some length certain arguments, to show that the operations with impossible quantities must have a logic equally strict and certain with the logic that appertains to real quantities, and that the aid obtained by these quantities would be perfectly useless if such conclusions rested only on the frail basis of analogy.

The author proceeds next to show that operations with imaginary quantities are by no means mechanical, but that they are conducted according to the rules of strict and rigorous logic; and that, although strictly speaking no proposition concerning them can be true or false, yet, after the demonstrations of certain formulæ for real quantities, demonstrations with impossible quantities may be legitimately and logically conducted. The series, for instance, for the development of an exponential, when the exponent is an impossible quantity, can never, independently of certain arbitrary assumptions, be duly established ; and yet, when the exponent is the sign of a real quantity, the formula for the development may be rigorously proved. With regard to demonstration, it is shown, as in the case of real quantities, it actually proceeds by a series of transformation, each proved to be the same as the foregoing, not by any arguments grounded on the properties of real quantities, but by reference to the forms certain abridged symbols are made to represent, and to the nature of the operations directed to be performed with certain signs.

After thus establishing the principle by which operations with imaginary characters are regulated, the author shows its efficacy and

the use of imaginary characters in the summation of series, proceeding according to the powers of the series and cosines of arcs in arithmetical progression. He likewise shows, that according to his mode of explanation, certain ambiguous expressions that occur in analysis are perfectly intelligible, and that in the controversy concerning the logarithms of negative quantities, carried on formerly between Leibnitz, Bernoulli, Euler, and Dalembert, all paradox and ambiguity may be made to disappear, by referring to the origin and real import of the impossible exponential quantities.

Although the principal object of this paper is to vindicate the indubitable justness of the operation conducted with imaginary characters, yet in the latter part some arguments are likewise offered in favour of the commodiousness of imaginary expressions for facilitating calculations. And, lastly, it is contended, that in the present state of analysis, these expressions are particularly useful in deducing certain conclusions, which without their aid could not be obtained without much difficulty.

On the Production of Artificial Cold by Means of Muriate of Lime.
By Mr. Richard Walker. Communicated by Henry Cavendish, Esq.
F.R.S. Read January 22, 1801. [Phil. Trans. 1801, p. 120.]

Mr. Walker, since his late communications to the Society on the best means of producing artificial cold, received intelligence that Mr. Lowitz, Professor of Chemistry at Petersburgh, had made some experiments, in which a neutral salt different from those he had himself used, and which is but little known or attended to, produced effects which exceeded his expectations. The salt is the muriated lime; which, mixed with snow in the proportion of about 3 to 2, at the temperature of $+27°$, produced a refrigeration which sunk the thermometer to $-55°$; and with this mixture the Professor in one experiment froze no less than 35lbs of quicksilver.

Mr. Walker repeated the experiment with success; but finding that it can only be made during a freezing atmosphere, he resolved to try the effect of this salt, reduced to such a strength by evaporation as to endure being kept in a solid state throughout the year. After describing the expedients used for this purpose, he enumerates the results of two sets of experiments; the first made with the muriated lime prepared so as to be used in winter only, that is, of the specific strength of $1\cdot450$; and the other made with the salt prepared so as to be kept for use at any time, the strength of which was $1\cdot490$. The apparatus here used (though somewhat improved) is not unlike that described in Mr. Walker's former communication, nor is the process materially different.

The paper concludes with a general view of the different frigorific mixtures:—1st, those composed of chemical substances with ice; and 2nd, those in which the use of ice is dispensed with. In a postscript the author suggests a method of obtaining transparent ice, fit for optical purposes, which is effected merely by immersing a vessel